

# Strong-coupling superconductivity beyond BCS and the key pairing interaction in cuprate superconductors

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It has been now over 20 years since the discovery of the first high temperature superconductor by Georg Bednorz and Alex Müller in 1986 and yet, despite intensive effort, no universally accepted theory exists about the origin of high-temperature superconductivity. A controversial issue on whether the electron-phonon interaction (EPI) is crucial for high-temperature superconductivity or weak and inessential has been one of the most challenging problems of contemporary condensed matter physics. I briefly review our recent theoretical results, which in conjunction with a great number of experimental observations including isotope effects, angle-resolved photoemission (ARPES), pump-probe and tunnelling spectroscopies, normal state diamagnetism and magnetic quantum oscillations provide the definite answer to this fundamental question. The true origin of high-temperature superconductivity is found in a significant finite-range Fröhlich EPI of *nonadiabatic* polaronic carriers which is beyond the conventional BCS-Migdal-Eliashberg approximation.

After we have shown [1]— unexpectedly for many researchers— that the BCS-Migdal-Eliashberg (BCS-ME) theory breaks down already at the EPI coupling  $\lambda \gtrsim 0.5$  for any adiabatic ratio  $\hbar\omega_0/E_F$ , the multi-polaron physics has gained particular attention [2]. The parameter  $\lambda\hbar\omega_0/E_F$ , which is supposed to be small in the BCS-ME theory becomes in fact large at  $\lambda \gtrsim 0.5$  since the electron band narrows and the Fermi energy,  $E_F$  turns out below the characteristic phonon energy,  $\hbar\omega_0$ . Nevertheless, as noted in the unbiased comment by Jorge Hirsch [3], in order to explain the increasingly higher  $T_c$ s found in supposedly conventional materials, values of the electron-phonon coupling constant  $\lambda$  larger than 1 have been used in the conventional BCS-ME formalism. This formalism completely ignores the polaronic collapse of the bandwidth, but regrettably continues to be used by some researchers (irrespective of whether  $\lambda$  is small or large) failing to explain high- $T_c$ . As a result, many researchers maintain that a repulsive electron-electron interaction is responsible for pairing providing high transition temperatures,  $T_c$ , without phonons in high-temperature superconductors.

Properly extending the BCS theory towards the strong interaction between electrons and ion vibrations, a *Bose liquid* of tightly bound electron pairs surrounded by the lattice deformation (i.e. of *small bipolarons*) was predicted [4]. Further prediction was that high temperature superconductivity should exist in the crossover region of the electron-phonon interaction strength from the BCS-like to bipolaronic superconductivity [1], Fig.1. The strong enhancement of  $T_c$  in the crossover region from BCS-like polaronic to BEC-like bipolaronic superconductivity is entirely due to a sharp increase of the density of states in a narrow polaronic band [1], which is missing in the so-called *negative* Hubbard  $U$  model.

We have recently quantified the EPI strength, the phonon-induced electron-electron attraction, and the carrier mass renormalization in layered superconductors at

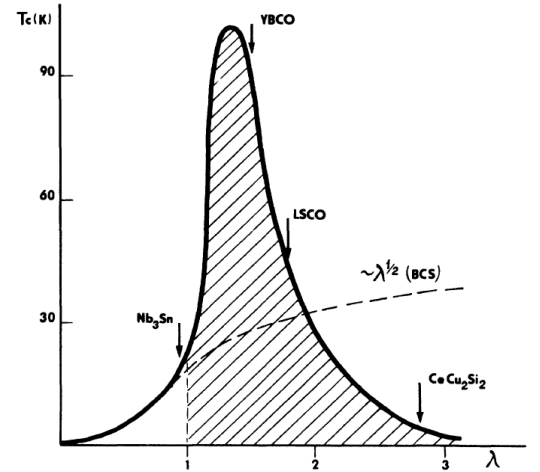


FIG. 1: Breakdown of the BCS-ME approximation due to the polaron bandwidth collapse at some critical coupling  $\lambda$  of the order of 1. The dependence of  $T_c$  versus  $\lambda$  is shown in the polaron-bipolaron crossover region compared with the BCS-ME dependence (dotted line). (Reproduced from Alexandrov A S 1988 *Phys. Rev. B* **38** 925, (©) American Physical Society, 1988.)

different doping using a continuum approximation for the renormalized carrier energy spectrum and the RPA dielectric response function [5]. If, for instance we start with a parent insulator as  $\text{La}_2\text{CuO}_4$ , the magnitude of the Fröhlich EPI is unambiguously estimated using the static,  $\epsilon_s$  and high-frequency,  $\epsilon_\infty$  dielectric constants. To assess its strength, one can apply an expression for the polaron binding energy (polaronic level shift)

$E_p$ , which depends only on the measured  $\epsilon_s$  and  $\epsilon_\infty$ ,  $E_p = (e^2/2\epsilon_0\kappa) \int_{BZ} d^3q/(2\pi)^3 q^2$ . Here, the integration goes over the Brillouin zone (BZ),  $\epsilon_0 \approx 8.85 \times 10^{-12}$  F/m is the vacuum permittivity, and  $\kappa = \epsilon_s\epsilon_\infty/(\epsilon_s - \epsilon_\infty)$ . In the parent insulator, the Fröhlich interaction alone provides the binding energy of two holes,  $2E_p$ , *an order of magnitude* larger than any magnetic interaction ( $E_p = 0.647$  eV in  $\text{La}_2\text{CuO}_4$ ).

Recent observations of the quantum magnetic oscillations in some cuprate superconductors [6] are opening up a possibility for a quantitative assessment of EPI in these and related doped ionic lattices with the quasi two-dimensional (2D) carrier energy spectrum. The oscillations revealed almost cylindrical Fermi surfaces, enhanced effective masses of carriers (ranging from  $2m_e$  to  $6m_e$ ) and the astonishingly low Fermi energy,  $E_F$ , which is well below 40 meV in Y-Ba-Cu-O. Such low Fermi energies make the Migdal-Eliashberg (ME) adiabatic approach to EPI inapplicable in these compounds since the characteristic oxygen vibration energy (about  $\hbar\omega_0 = 80$  meV) turns out larger than the carrier kinetic energy. Since carriers in cuprates are in the non-adiabatic (underdoped) or near-adiabatic (overdoped) regimes,  $E_F \lesssim \hbar\omega_0$ , their energy spectrum renormalized by EPI and the polaron-polaron interactions can be found with the familiar small-polaron canonical transformation at *any coupling*  $\lambda$  [7].

With doping the attraction and the polaron mass drop [5]. Nevertheless, on-site and inter-site attractions induced by EPI remain several times larger than the superexchange (magnetic) interaction  $J$  (about 100 meV) at any doping since the non-adiabatic carriers cannot fully screen high-frequency electric fields. The polaron mass [5] agrees quite well with the experimental masses measured in magnetic quantum oscillations experiments. Hence the Fröhlich EPI with high-frequency optical phonons turns out to be the key pairing interaction in underdoped cuprates and remains the essential player at overdoping. What is more surprising is that EPI is clearly beyond the BCS-ME approximation since

its magnitude is larger than or comparable with the Fermi energy and the carriers are in the non-adiabatic or near-adiabatic regimes. Since EPI is not local in the nonadiabatic electron system with poor screening it can provide the d-wave symmetry of the pairing state [8].

There are other independent pieces of evidence in favor of (bi)polarens and 3D BEC in cuprate superconductors. Most compelling evidence for (bi)polarens in cuprate superconductors is the substantial isotope effect on the carrier mass [9]. High resolution ARPES [10] provides another piece of evidence for a strong electron-phonon interaction (EPI) in cuprates and related oxides apparently with c-axis-polarised optical phonons. These as well as recent pump-probe experiments [11] unambiguously show that the Fröhlich EPI is important in those highly polarizable ionic lattices.

Magnetotransport data strongly support preformed bosons in cuprates. In particular, many high-magnetic-field studies revealed a non-BCS upward curvature of the upper critical field  $H_{c2}(T)$ , predicted for the Bose-Einstein condensation of charged bosons in the magnetic field [12]. Nonlinear normal-state diamagnetism of quite a few hole-doped cuprates has a profile characteristic of normal state real-space composed bosons (i.e. bipolarons) [13], rather than "preformed" Cooper pairs, vortex liquid and the Kosterlitz-Thouless phase transition hypothesized by some authors.

Single polarons, *localised* within an impurity band-tail, coexist with bipolarons in charge-transfer doped Mott-Hubbard insulators. They account for sharp "quasi-particle" peaks near  $(\pi/2, \pi/2)$  of the Brillouin zone and high-energy "waterfall" effects observed with ARPES in cuprate superconductors [14]. This "band-tail" model also accounts for two energy scales (superconducting and pseudo-gaps) in ARPES [14] and in the extrinsic and intrinsic tunnelling [15].

All these and many other observations point to a crossover from the bipolaronic to polaronic superconductivity [1] in high-temperature superconductors with doping.

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